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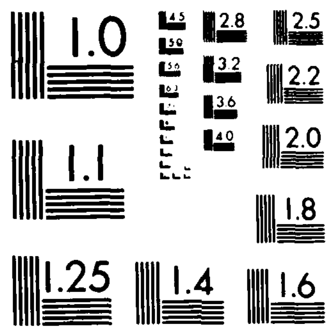

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TECHNICAL REPORT GL-86-15

PROBABILISTIC AND RELIABILITY ANALYSIS OF THE CALIFORNIA BEARING RATIO (CBR) DESIGN METHOD FOR FLEXIBLE AIRFIELD PAVEMENTS

by

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The California Bearing Ratio (CBR) design method for flexible airfield pavements was analyzed using a probabilistic approach. The design parameters considered were the load P (or the equivalent single-wheel load), the subgrade CBR, the tire contact area A, and the pavement total thickness t. The expected value and variance of the dependent variable performance factor α (which is logarithmically related to the number of passes to failure) were estimated by using the Taylor series expansion and the Rosenblueth method.
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20. ABSTRACT (Continued).

Differences in computed results between the two methods were found to be small, although the derivation of the expressions for Taylor series expansion was very complicated. A procedure was developed to estimate the reliability of the designed pavement system based on known variabilities of design parameters. Results of the reliability analysis indicate that prediction of pavement performance is most influenced by variations of pavement thickness t and is least influenced by variations of tire contact area A . The effects of variations of wheel load P and subgrade CBR are identical. The weighting factors for parameters t , CBR , P , and A , in general, are approximately 1, 0.34, 0.34, and 0.01, respectively (Table 3).

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PREFACE

The work reported herein was funded by the Office, Chief of Engineers (OCE), US Army, under the FY 82 RDTE Program, Project: 4A161102AT22, Task AO, Work Unit 009, "Methodology for Considering Material Variability in Pavement Design." Mr. S. L. Gillespie was the Technical Monitor for OCE.

The study was conducted by the US Army Engineer Waterways Experiment Station (WES), Geotechnical Laboratory (GL). Dr. W. F. Marcuson III was Chief, GL, and Mr. H. H. Ulery was Chief, Pavement Systems Division (PSD), during this period. Dr. Yu T. Chou, PSD, was in charge of the study and wrote this report. The report was edited by Ms. Odell F. Allen of the WES Information Products Division.

Col Allen F. Grum, USA, was the previous Director of WES. COL Dwayne G. Lee, CE, is the present Commander and Director. Dr. Robert W. Whalin is Technical Director.

CONTENTS

CONVERSION FACTORS, NON-SI TO SI (METRIC) UNITS OF MEASUREMENTS

Non-SI units of measurement used in this report can be converted to SI (metric) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
inches	2.54	centimetres
pounds (force)	4.448222	newtons
pounds (force) per square inch	6.894757	kilopascals
square inches	6.4516	square centimetres

PROBABILISTIC AND RELIABILITY ANALYSIS OF THE CALIFORNIA BEARING RATIO (CBR) DESIGN METHOD FOR FLEXIBLE AIRFIELD PAVEMENTS

PART I: INTRODUCTION

Background

1. The design of flexible airfield pavements in the US Army Corps of Engineers is based on two methods: (a) the California Bearing Ratio (CBR) equation which is empirical in nature and yields a design thickness for a given design condition, and (b) the multilayered elastic system which is analytical in nature and yields stresses, strains, and deflections in the pavement system for a particular loading condition and pavement geometry which in turn are compared to established failure criteria to determine the performance of the given pavement. Both of the design method approaches are deterministic, i.e., a unique pavement system is designed for the unique set of input variables. For instance in the CBR method, a given pavement thickness is determined from given values of subgrade CBR , gear load and configuration, tire contact area, and design coverage level. The effect of material variability on pavement performance is considered in the designer's selection of the subgrade CBR value, and the design safety factor is inherent in the construction specifications such as compaction requirements. However, a quantification of these effects can be accomplished by using the probabilistic approach, and the design procedures can be improved by showing the partial effect of each design parameter.

Purpose and Scope

2. The purpose of this study is to investigate the partial effect of the variability of design parameter in the CBR equation on pavement performance. The effects of each parameter are to be quantitatively evaluated on the final design. The final design will be expressed in terms of probability and reliability, stressing the crucial parameters which should be tightly controlled in the construction phases, and/or the crucial loading parameters

dictated by the intended use of the pavement.

3. The methodologies used in the probabilistic analysis of the CBR equation are the Taylor series expansion and the Rosenblueth method. Computer programs were developed for both methods and example pavements were analyzed. Procedures are presented to determine the reliability of pavement as a function of variabilities of design parameters.

PART II: PROBABILISTIC APPROACH

General

4. The original and the new CBR equation for flexible airfield pavements are shown below as Equations 1 and 2, respectively.

$$t = \alpha \sqrt{\frac{P}{8.1\ CBR} - \frac{A}{\pi}} \tag{1}$$

$$t = \alpha \left\{ \sqrt{A} \left[-0.0481 - 1.1562 \left(\log \frac{CBR \cdot A}{P} \right) - 0.6414 \left(\log \frac{CBR \cdot A}{P} \right)^2 - 0.473 \left(\log \frac{CBR \cdot A}{P} \right)^3 \right] \right\} \tag{2}$$

where

t = pavement thickness

A = tire contact area

CBR = California Bearing Ratio of the subgrade soil

P = single-wheel load (or the equivalent single-wheel load (ESWL) in the case of the multiple-wheel loads)

α = a traffic factor equal to or less than one (see Figure 1)

Equation 1 was formulated in the 1950's (Fergus 1950, US Army Waterways Experiment Station 1951, and Turnbull and Ahlvin 1957), and Equation 2 is the new form based on more test data formulated in the early 1970's (Hammitt, et al. 1971).

5. Since Equations 1 and 2 provide the unique flexible pavement thickness t, for unique values of load P, subgrade CBR, and tire contact area A, they are completely determined or DETERMINISTIC. However, no direct consideration and evaluation in these equations are made for the effect of the parameter variabilities on the design. Since the subgrade soil is not uniform under an airfield and the magnitude of one aircraft loading, as well as the tire contact area, is different from the others, the natural variation of each design parameter will have certain impact on the performance of the finished pavement and the actual magnitude of the variation of each parameter will also differ among the design parameters. For instance, the variation of the subgrade soil is greater than that of the pavement thickness as the latter is

easier to control during construction. Also, the relative effects of these variations on pavement performance could be vastly different; the effect of thickness variations may be much greater than tire contact area variations. The deterministic design approach does not directly take care of these variabilities and thus does not provide information for decision making in varying situations of design and construction. As an inherent part of the system, parameter variabilities should be included in the design procedure in a quantitative manner to provide a more rational tool for the designer.

6. The methods to calculate the expected value and variance of a function (as the α function in Equation 1) using the Taylor series expansion (Benjamin and Cornell 1970) and the Rosenblueth procedure (1975), are presented below, followed by the procedure to compute the reliability of the design.

Taylor Series Expansion

7. The Taylor formula for the expansion of a function $f(x)$, with N continous derivatives, about the mean μ is

$$f(x) = f(\mu) + f'(\mu)(x - \mu) + \frac{f''(\mu)}{2}(x - \mu)^2$$

$$+ \ldots \text{ higher order terms } + \text{ remainder} \qquad (3)$$

Since the expected value of $(x - \mu)$ is zero and the expected value of $(x - \mu)^2$ is the variance* of x, i.e., $E(x - \mu) = 0$ and $E(x - \mu)^2 = \sigma_x^2$, the expected value of $f(x)$ becomes

$$E[f(x)] = f(\mu) + 0 + \frac{1}{2} f''(\mu)\, \sigma_x^2 + \ldots..$$

$$E[f(x)] \approx f(\mu) + \frac{1}{2} f''(\mu)\, \sigma_x^2 \qquad (4)$$

The expected value of $f^2(x)$ is expressed as

---

* Definitions of expectation and variance are presented in Appendix A.

$$
\begin{aligned}
E\left[f^2(x)\right] &\approx f^2(\mu) + \frac{1}{2}\,[f^2(\mu)]''\sigma_x^2 \\
&\approx f^2(\mu) + \frac{1}{2}\left[2f(\mu)f'(\mu)\right]'\sigma_x^2 \\
&\approx f^2(\mu) + \frac{1}{2}\left[2f'(\mu)f'(\mu) + 2f(\mu)f''(\mu)\right]\sigma_x^2 \\
&\approx f^2(\mu) + \ [f'(\mu)]^2 + f(\mu)\ f''(\mu)\ \sigma_x^2
\end{aligned}
\tag{5}
$$

The variance of a variable x is derived as follows:

$$
\begin{aligned}
\sigma_x^2 &= V\left[x\right] = E\left[(x - \mu)^2\right] = E\left[x^2 - 2\mu x + \mu^2\right] \\
&= E\left[x^2\right] - 2\mu\ E\left[x\right] + E\left[\mu^2\right]
\end{aligned}
$$

as $E\left[x\right] = \mu$; and $E\left[\mu^2\right] = \mu^2$, as μ is a constant,

$$
\sigma_x^2 = V\left[x\right] = E\left[x^2\right] - 2\mu^2 + \mu^2 = E\left[x^2\right] - \left[E(x)\right]^2 \tag{6}
$$

In other words, the variance is said to be the mean square minus the square mean. The variance of $f(x)$ can be written as

$$
V\left[f(x)\right] = E[f^2(x)] - \ E\left[f(x)\right]^{\,2} \tag{7}
$$

Then substituting Equations 4 and 5 into Equation 7 results in

$$
\begin{aligned}
V\left[f(x)\right] &\approx f^2(\mu) + \ [f'(\mu)]^2 + f(\mu)\ f''(\mu)\ \sigma_x^2 \\
&\quad - \ f^2(\mu) + f(\mu)\ f''(\mu)\ \sigma_x^2 + \frac{1}{4}\left[f''(\mu)\right]^2\ \sigma_x^4
\end{aligned}
\tag{8}
$$

$$
V[f(x)] = \left[f'(\mu)\right]^2\ \sigma_x^2 - \frac{1}{4}\left[f''(\mu)\right]^2\ \sigma_x^4
$$

8. In Equations 4 and 8, if the random variables can be assumed normally distributed, the second-order terms may be neglected. For multivariate situations, the first-order approximation to the expectation and the variance of $f(x)$ is expressed by Benjamin and Cornell (1970) as

$$E\left[f(x)\right] = f(\mu) \tag{9}$$

$$V\left[f(x)\right] = \sum_{i=1}^{N} \sum_{j=1}^{N} \left(\frac{\partial f}{\partial x_i}\bigg|_{\text{all } \bar{x}_i} \frac{\partial f}{\partial x_j}\bigg|_{\text{all } \bar{x}_i} \right) \text{Cov } (x_i, x_j) \tag{10}$$

where Cov (x_y , x_j) is the covariance of variable x_i and x_j . Note that if the x_i are uncorrelated, Equation 10 is simply

$$V\left[f(x)\right] = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\bigg|_{\text{all } \bar{x}_i} \right)^2 V(x_i) \tag{11}$$

Rosenblueth Method

9. Equations 4 and 8 are obtained from the Taylor expansion of the function about the expectations of the random variables. This method requires the existence and continuity of the first and second derivatives of the function. Rosenblueth (1975) overcame these difficulties through use of point estimates of the function. The expressions for the expected value are:

$$E\left[\alpha^N\right] = \frac{1}{2}\left(\alpha_+^N + \alpha_-^N\right) \qquad \text{for one variable} \tag{12}$$

$$E\left[\alpha^N\right] = \frac{1}{2^2}\left(\alpha_{++}^N + \alpha_{+-}^N + \alpha_{-+}^N + \alpha_{--}^N\right) \qquad \text{for two variables} \tag{13}$$

$$E\left[\alpha^N\right] = \frac{1}{2^3}\left(\alpha_{+++}^N + \alpha_{++-}^N + \alpha_{+--}^N + \alpha_{+-+}^N + \alpha_{-++}^N + \alpha_{-+-}^N + \alpha_{--+}^N + \alpha_{---}^N\right) \qquad \text{for three variables} \tag{14}$$

$$E[\alpha^N] = \frac{1}{2^4}\Big(\alpha^N_{++++} + \alpha^N_{+++-} + \alpha^N_{++-+} + \alpha^N_{++--} + \alpha^N_{+-++}$$

$$+ \alpha^N_{+-+-} + \alpha^N_{+--+} + \alpha^N_{+---} + \alpha^N_{-+++} + \alpha^N_{-++-}$$

$$+ \alpha^N_{-+-+} + \alpha^N_{-+--} + \alpha^N_{--++} + \alpha^N_{--+-} \tag{15}$$

$$+ \alpha^N_{---+} + \alpha^N_{----}\Big) \text{ for four variables}$$

$$E[\alpha^N] = \frac{1}{2^M}\Big(\alpha^N_{\underbrace{+++++++}_{M}} + \ldots + \alpha^N_{\underbrace{-------}_{M}}\Big) \text{ for M variables} \tag{16}$$

Note that the number of total terms to calculate the expected value of a function y which has M variables is 2^M and N have a value of either 1 or 2 as shown in Equations 6 and 7.

10. To illustrate the use of the Rosenblueth method, the expected value of the α factor in Equation 2 is calculated. Since Equation 2 has four independent parameters, t, CBR, A, and P, Equation 15 is used to determine the expected value of the α factor. Assuming that the standard deviations of the parameters are σ_t, σ_{CBR}, σ_A, and σ_P and that the parameters are arranged in the order of t, CBR, A, and P (i.e., the order of the symbols $++++$, $+++-$, ..., etc.), the terms in Equation 15 can be written as

$$\alpha^N_{++++} = \frac{(t + \sigma_t)}{\sqrt{A + \sigma_A}}\Bigg\{-0.0481 - 1.1562\left[\log \frac{(CBR + \sigma_{CBR})(A + \sigma_A)}{(P + \sigma_P)}\right]$$

$$- 0.6414\left[\log \frac{(CBR + \sigma_{CBR})(A + \sigma_A)}{(P + \sigma_P)}\right]^2 - 0.473\left[\log \frac{(CBR + \sigma_{CBR})(A + \sigma_A)}{(P + \sigma_P)}\right]^3\Bigg\}$$

$$\vdots$$

$$\alpha^N_{-+++} = \frac{(t - \sigma_T)}{\sqrt{A + \sigma_A}}\Bigg\{-0.0481 - 1.1562\left[\frac{(CBR + \sigma_{CBR})(A + \sigma_A)}{(P - \sigma_P)}\right]$$

$$- 0.6414\left[\log \frac{(CBR + \sigma_{CBR})(A + \sigma_A)}{(P + \sigma_P)}\right]^2 - 0.473\left[\log \frac{(CBR + \sigma_{CBR})(A + \sigma_A)}{(P + \sigma_P)}\right]^3\Bigg\}$$

$$\vdots$$

$$\alpha^N_{----} = \frac{(t - \sigma_T)}{\sqrt{A - \sigma_A}}\Bigg\{-0.0481 - 1.1562\left[\log \frac{(CBR - \sigma_{CBR})(A - \sigma_A)}{(P - \sigma_P)}\right]$$

$$- 0.6414\left[\log \frac{(CBR - \sigma_{CBR})(A - \sigma_A)}{(P - \sigma_P)}\right]^2 - 0.473\left[\log \frac{(CBR - \sigma_{CBR})(A - \sigma_A)}{(P - \sigma_P)}\right]^3\Bigg\} \tag{17}$$

11. If the mean values for parameters t , CBR , A , and P and their standard deviations σ_t , σ_{CBR} , σ_A , and σ_P are known, the expected value of α can be estimated from Equations 15 and 17, and the variance of α is computed by Equation 7.

Reliability Analysis

12. As soon as the expected value and the variance of a function (such as the α factor in Equation 2 representing the design performance level) are determined, the reliability level of the function can be computed. Reliability is defined to be the probability that the pavement system will perform its intended function over its design life and under the conditions encountered during operation (Darter and Hudson 1973). The procedure to follow is explained below.

13. The relationship between the load repetition factor α and the aircraft passes is shown in Figure 1 (Hammitt et al. 1971). The problem existing at this stage is to determine the reliability level of the available design curves and the design equations. If the design curve is drawn through test data points at failure with no consideration of the safety factor (i.e., 50 percent of the data points above the curve and the other 50 percent below the curve), the reliability of such a design is 0.5. In other words, the probability of success of this design is only 50 percent. However, if a certain amount of safety factor has been considered in the design curves or the design equation, the reliability of the design should be greater than 0.5. In a recent study (Potter 1985), reliability of the CBR equation was determined to be 0.5 without including the effects of conservative estimates for the parameters of material strength, traffic load, and traffic intensity. Assumed reliability values are used in the illustrated computations.

14. With the load repetition factor α assumed normally distributed, the number of aircraft passes corresponding to $\alpha[1 + C \cdot CV(\alpha)]$ can be determined from Figure 1, and the probability of $\alpha \leq \alpha[1 + C \cdot CV(\alpha)]$ is taken from the normal distribution. $CV(\alpha)$ is the coefficient of variation of α , which is the ratio of the standard deviation of α to a mean of α , (i.e., $\sigma_\alpha/\bar{\alpha}$), and C is a selected number varying from -3 to +3. C values less than -3 and greater than +3 are not necessary because the area under a normal distribution curve beyond -3 and +3 standard deviations are negligible. The

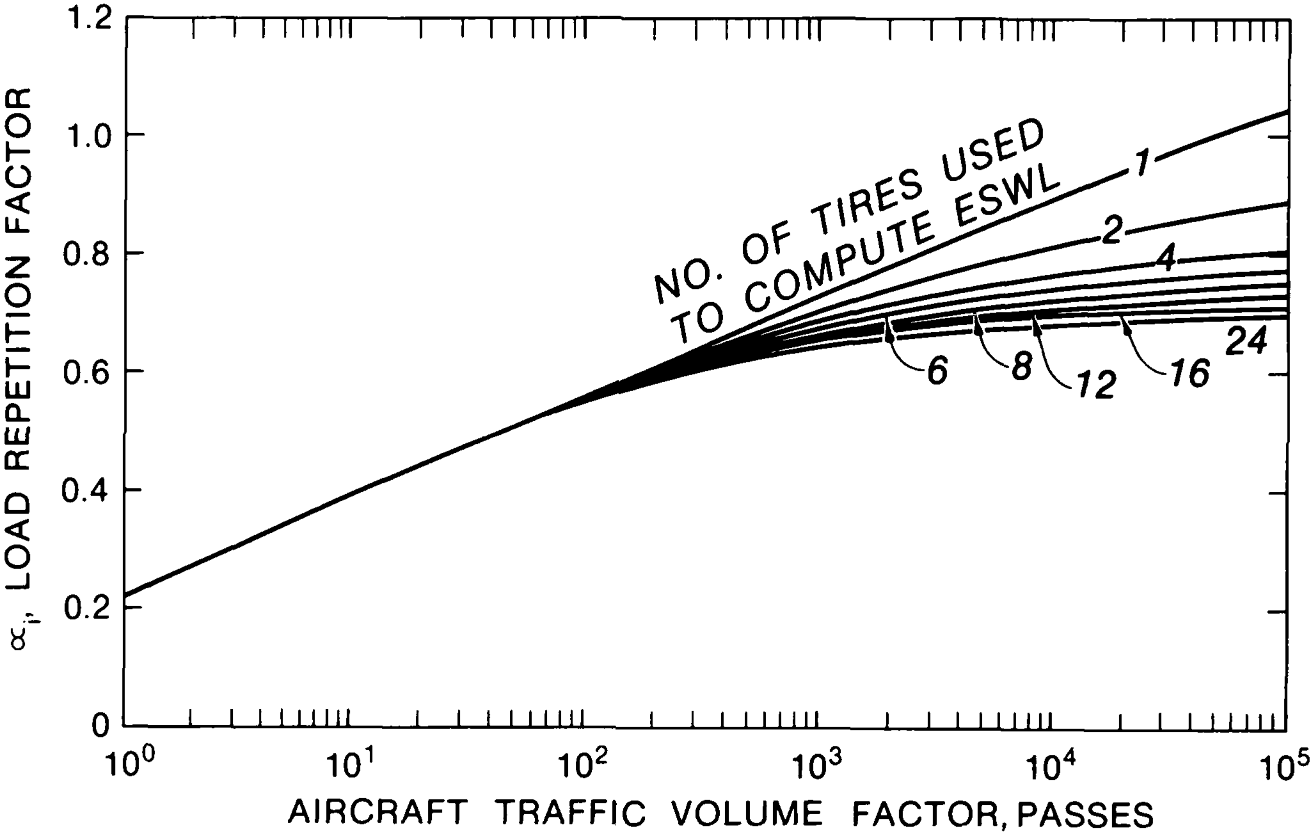

Figure 1. Composite plot of load repetition factors versus passes

computations of the reliabilities, α factors, and aircraft passes for given airfield pavements are illustrated in the following example.

Assumed conditions

15. The mean values of the single-wheel load P (or the ESWL), subgrade CBR , and tire contact area A are assumed to be 30,000 lb,* 4, and 285 sq in., respectively. A series of mean pavement thickness t = 10, 12, 15, 17.5, 20, 22.5, 25 in. are used. The coefficients of variation of the four parameters are all assumed to be 10 percent. For instance, if the mean wheel load is 30,000 lb, the standard deviation of the wheel load will be 3,000 lb, i.e., 68.3 percent of the time the wheel load would lie between 27,000 to 33,000 lb.

Computations

16. Computer programs were prepared for the calculations of expected values and variances derived from the Taylor series expansion and Rosenblueth method. Unless otherwise noted, results presented in this report are computed using the Taylor series expansion method.

17. For a pavement thickness t = 20 in., the computed (mean) α value using Equation 2 is 0.72, and the variance of the α value (i.e., $V(\alpha)$) computed using Equations 6 and 15 is 0.0095739. The standard deviation of the α value (σ_α) is $\sqrt{0.0095739} = 0.09785$ (or 0.1). Table 1 shows the computed passes for an assumed reliability values of 0.7 and 0.5 of the CBR equation.

18. The reliabilities are calculated from the normal distribution and the aircraft passes are obtained from the relationships in Figure 1 for the single-wheel load case. Similar computations can also be made for other pavement thicknesses (i.e., t = 10, 12, 15, 17.5, 20, 22.5, 25 in.). The results shown in Table 1 and those computed for other thicknesses can be plotted in Figure 2 and Figure 3, for initial reliabilities of 0.7 and 0.5, respectively, showing the relationships between reliability levels and aircraft passes for varying pavement thicknesses. Note that the procedures presented in this report are applicable for any reliability value of the CBR equation. A comparison of Figures 2 and 3 reveals that the patterns of the curves in each are similar. The significance of the curves will be discussed in Part III.

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* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.

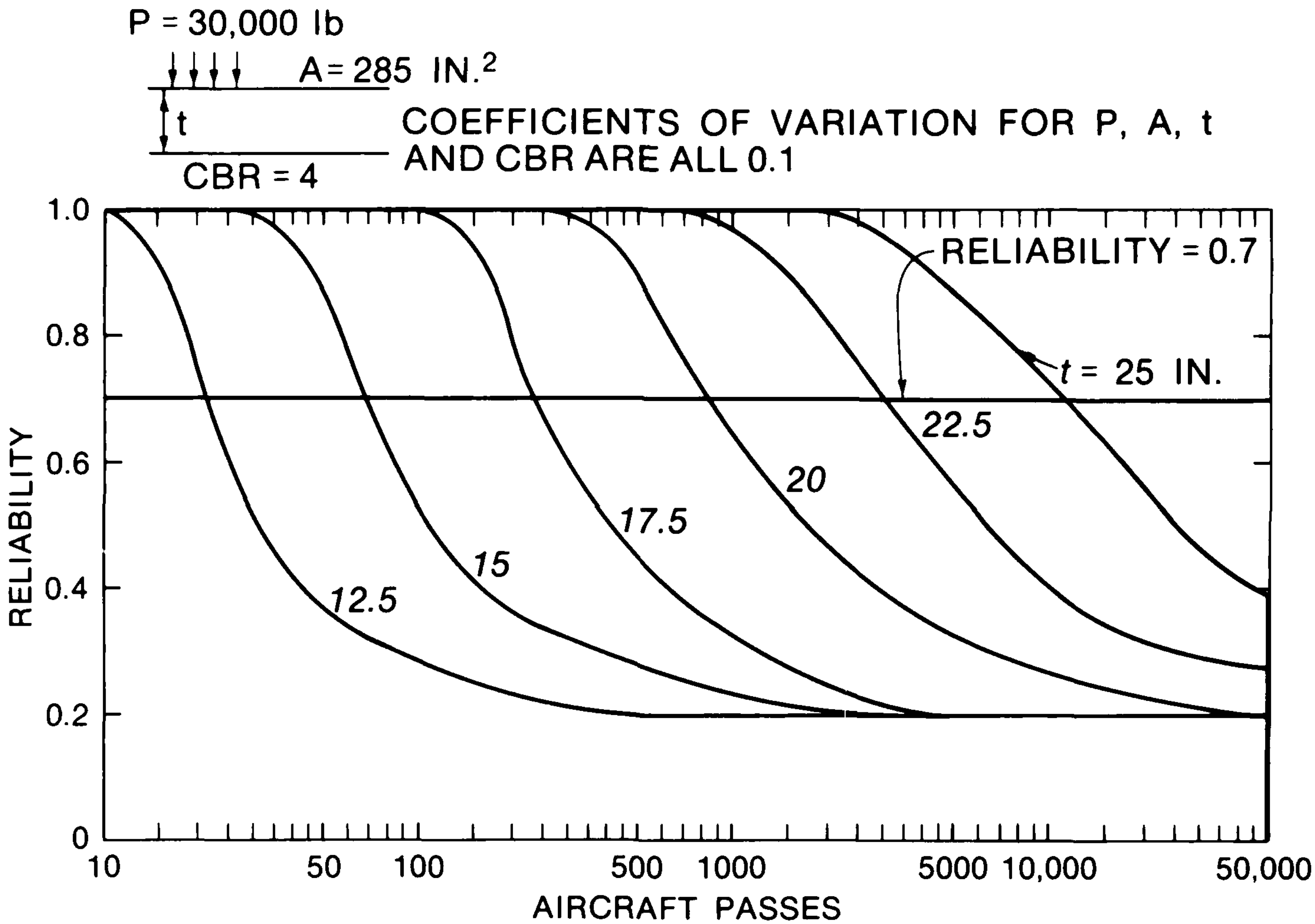

Figure 2. Relationships between reliability and aircraft passes for varying pavement thicknesses, reliability of CBR equation equals 0.7

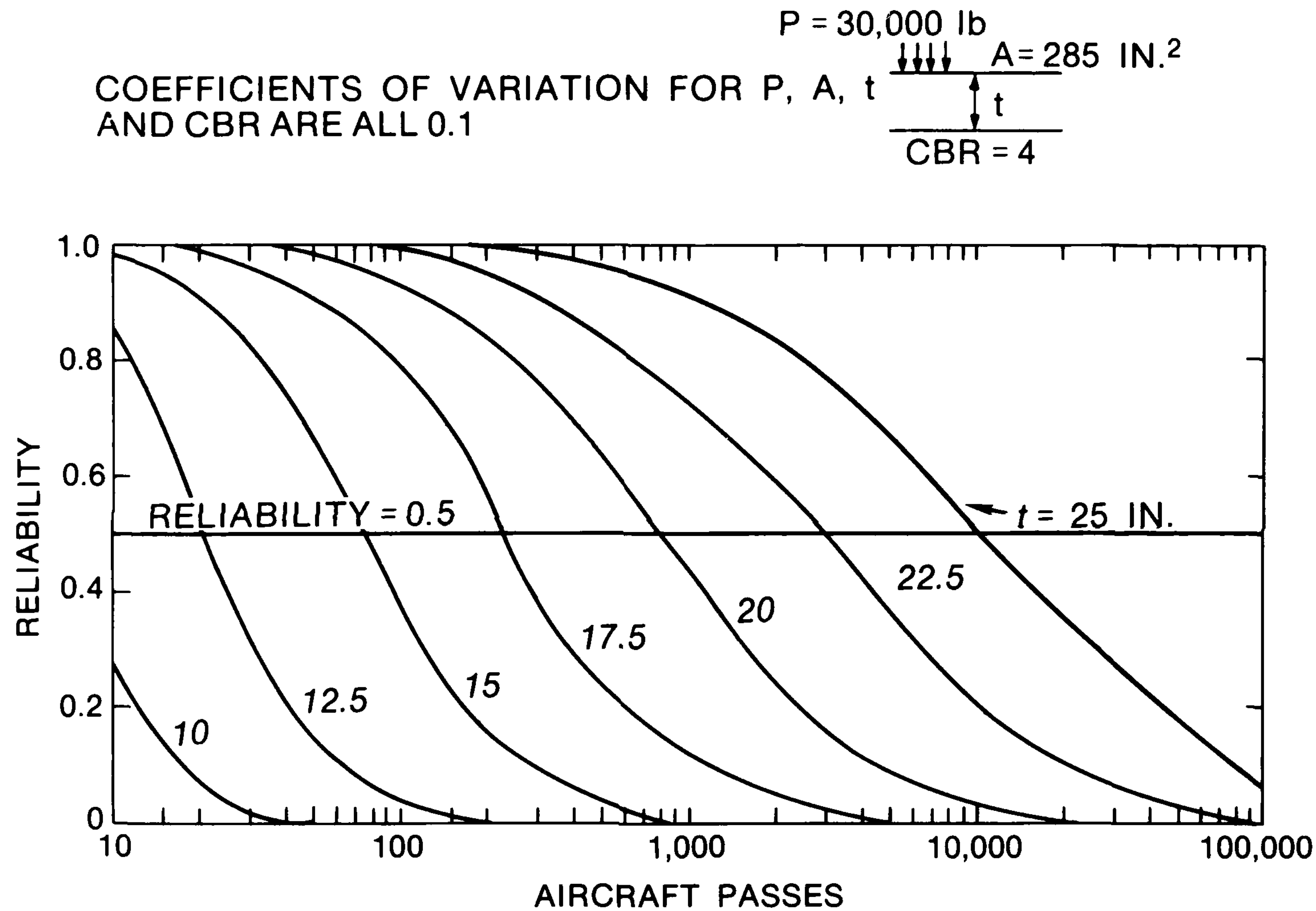

Figure 3. Relationships between reliability and aircraft passes for varying pavement thicknesses, reliability of CBR equation equals 0.5

PART III: ANALYSIS OF THE CBR EQUATION

Development of the CBR Equation

19. Formulation of the CBR equation (Turnbull and Ahlvin 1957) was based on results of numerous full-scale accelerated traffic tests, which represented the reliable data and extensive observations accumulated by the Corps of Engineers. The theory of elasticity was also used to formulate the equation. It was found from the theory of elasticity that for a given intensity of surface load the stresses beneath total loads of different magnitudes would be equal at homologous points. It was assumed that the needed strength and therefore the required CBR would be the same at depths at which the stresses are identical. Thus, for a given CBR and intensity of load, the depth of cover or thickness of protective layer for any magnitude of load must be such that the ratio of the thickness t, to the radius of contact area r, is a constant, C, or

$$\frac{t}{r} = C \tag{18}$$

By using the relation $P = p\pi r^2$, where P and p are load and load intensity, respectively, the following equation resulted:

$$t = K\sqrt{P} \tag{19}$$

where $K = C/\sqrt{p\pi}$ is a constant dependent on the CBR.

20. Equation 19 relates the total load P to the pavement thickness t. The tire contact pressure p was not considered at the time. However, it was soon realized that Equation 19 was good only for tire pressure around or less than 100 psi. With the knowledge that an increase in tire pressure would require an increase in protective thickness (depth) such that the theoretical deflections would be equal, CBR relations were developed for higher tire pressures. The theoretical equation to compute the deflections and the resulted relation are given as

$$w = 1.5\,\frac{p}{E_m}\left(\frac{r^2}{\sqrt{r^2 + t^2}}\right) \tag{20}$$

and

$$D = K^2 + \frac{1}{p\pi} \tag{21}$$

where w and E_m are deflection and modulus of elasticity of the soil and D and K are constants.

21. Values for the constant K were developed from single-wheel CBR curves for design (or evaluation) of flexible airfield pavement for capacity operation (5,000 coverages). A relationship was found between D and the CBR ; that is, the product $D \times CBR$ was substantially constant for CBR values below about 10 to 12 and the value was about equal to 1/8.1. Based on this relation and Equation 21, the CBR equation (Equation 1) resulted.

22. It should be noted that in the CBR equation the required thickness is proportional to the parameter P/CBR , i.e., if the load is increased, the same thickness of pavement can be used as long as the subgrade CBR value is increased by the same proportion. Thus, Equations 1 and 2 can be written in the following forms:

$$\alpha = \frac{t}{\sqrt{\frac{P}{8.1\ CBR} - \frac{A}{\pi}}} \tag{22}$$

$$\alpha = A \Bigg/ \left\{ \sqrt{A} \cdot \left[-0.0481 - 1.1562 \left(\log \frac{CBR \cdot A}{P} \right) - 0.6414 \left(\log \frac{CBR \cdot A}{P} \right)^2 - 0.473 \left(\log \frac{CBR \cdot A}{P} \right)^3 \right] \right\} \tag{23}$$

23. For the discussion in this paragraph, Equation 22 is used because of its simplicity. The equation shows that the performance factor of the designed pavement α is proportional to the selected thickness of the pavement t and to the quantities CBR/P and A . The performance of a given pavement is improved with an increasing CBR value of the subgrade and the tire contact area A and is worsened as the aircraft load is increased. Also, as shown by Equation 22, if the load P is increased, the performance α can be kept unchanged if the subgrade CBR is also increased the same proportion; i.e., if the subgrade CBR is doubled, the load could be doubled. For example, the predicted performance is about 91 passes (Figure 1) for a 20-in. pavement and a 4-CBR subgrade subjected to a 30,000-lb wheel load. If another airfield runway designed for a 60,000-lb wheel load is to be built for

the same subgrade soil and if the same pavement thickness is used, then the same performance level can be expected if the subgrade CBR value is increased from 4 to 8 through stabilization. Although this conclusion is derived solely from the CBR equation, it is generally true based on actual field experiences.

Sensitivity Analysis

24. A sensitivity analysis was made for the CBR Equation 23 to examine the effect of variations of each parameter (i.e., t, P, CBR , and A) on pavement performance (α factor). In the analysis, the value of one parameter was varied each time while the other parameters were kept constant and values of the α factor were computed. To vary the value of each parameter, a multiplying factor ranging from 0.5 to 1.5 was used for each one. The base values for the parameters t, P, CBR , and A were 20-in., 30,000 lb, 4, and 285 sq in., respectively. Table 2 shows the different α values computed from Equation 23 for a series of wheel loads P. Various new P values resulted from the use of multiplying factors.

25. The relationship between α factor and the multiplying factor shown in Table 2 for the load P is plotted in Figure 4, together with other similar relationships computed for thickness t, subgrade CBR , and area A. The dotted line shown in the figure is horizontal for an α factor of 0.716, which is the α factor with the multiplying factor being equal to 1.0. All four curves plotted cross this line at the point where the multiplying factor has a value of 1.0. The slope of the curve indicates the degree of sensitivity of the performance factor α to the particular design parameter. The pavement performance (α factor) is most sensitive to the variation of pavement thickness t and is least sensitive to the tire contact area A. The effects of variations of the load P and the subgrade CBR on pavement performance are nearly the same, except that the effects are in the opposite direction. The reason that pavement performance is equally sensitive to the variations of load P and subgrade CBR is manifested in CBR Equation 1 in which the required pavement thickness t is proportional to the parameter P/CBR .

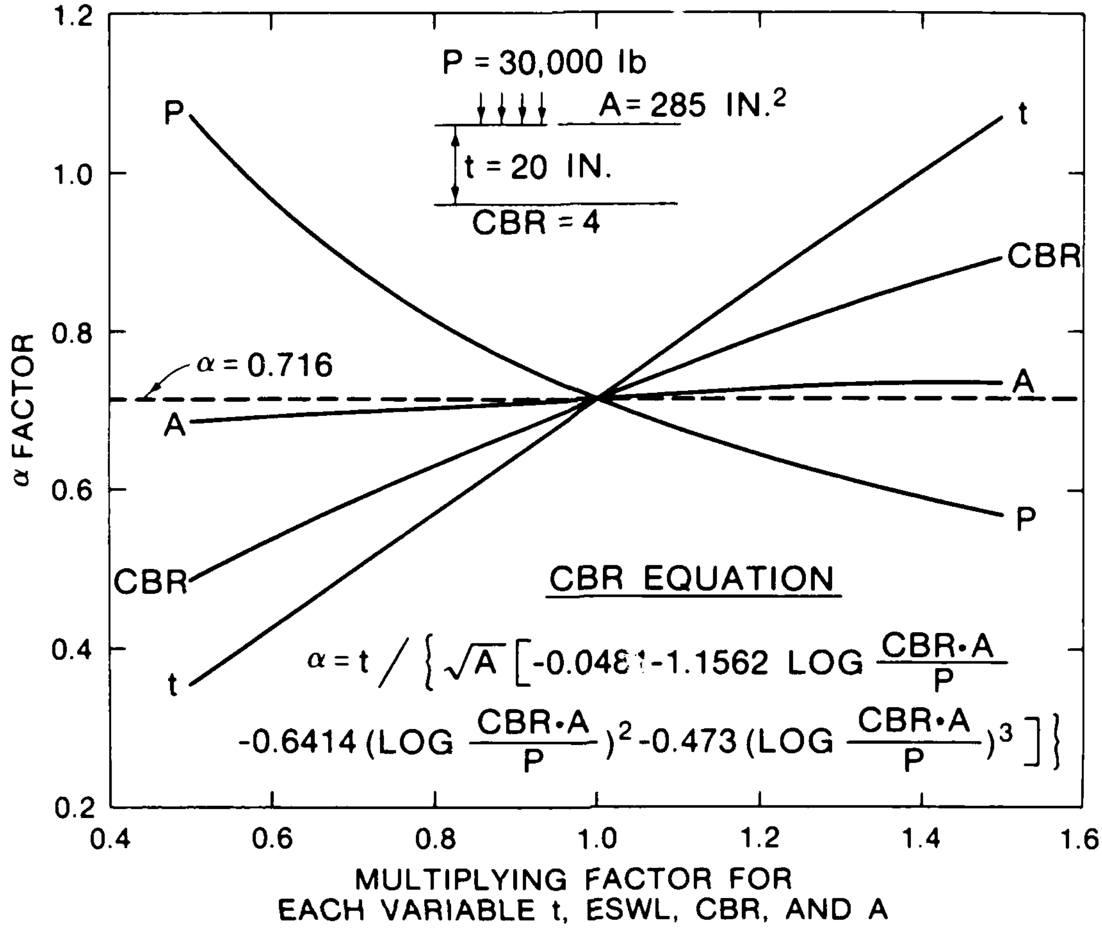

Figure 4. Sensitivity analysis of the CBR equation

Probabilistic Approach

26. Equation 8 shows the expression for variance of a function $f(x)$. The variance for the performance function α in the CBR equation is given in Equation B5 (Appendix B). By neglecting the nonlinear terms* and using the relations $V(\alpha) = \sigma^2(\alpha) = \bar{\alpha}^2 \cdot CV^2(\alpha)$ and $\alpha = t/D$, (Equation B3 in Appendix B), the following equation can be written from Equation B5.

$$CV^2(\alpha) = W_t CV^2(t) + W_A CV^2(A) + W_{CBR} CV^2(CBR) + W_p CV^2(p) \tag{24}$$

where

$CV^2(\alpha), CV^2(t), CV^2(A), CV^2(CBR), CV^2(p)$ = the square of the coefficient of variation of α, t, A, CBR, and P, respectively.

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* The calculated values of the nonlinear terms are insignificant (see paragraph 42.

$$W_t = 1$$

$$W_A = \frac{\bar{A}}{\bar{D}}\left[\frac{C_1 + 2C_2C_5}{2\sqrt{\bar{A}}} + \frac{C_2 + 4C_3C_5}{2\sqrt{\bar{A}}}\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right) + \frac{C_3 + 6C_4C_5}{2\sqrt{\bar{A}}}\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right)^2 + \frac{C_4}{2\sqrt{\bar{A}}}\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right)^3\right]^2$$

$$W_{CBR} = \frac{\overline{CBR}}{\bar{D}}\left[\frac{C_2C_5\sqrt{\bar{A}}}{\overline{CBR}} + \frac{2C_3C_5\sqrt{\bar{A}}}{\overline{CBR}}\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right) + \frac{3C_4C_5\sqrt{\bar{A}}}{\overline{CBR}}\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right)^2\right]^2$$

$$W_P = \frac{\bar{P}}{\bar{D}}\left[-\frac{C_2C_5\sqrt{\bar{A}}}{\bar{P}} - \frac{2C_3C_5\sqrt{\bar{A}}}{\bar{P}}\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right) - \frac{3C_4C_5\sqrt{\bar{A}}}{\bar{P}}\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right)^2\right]^2$$

$$\bar{D} = \sqrt{\bar{A}}\left[C_1 + C_2\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right) + C_3\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right)^2 + C_4\left(\log\frac{\overline{CBR}\cdot\bar{A}}{\bar{P}}\right)^3\right]^2$$

$\bar{t},\bar{A},\overline{CBR},\bar{P}$ = mean values of pavement thickness t, tire contact area A, subgrade CBR, and load P, respectively.

C_1,C_2,C_3,C_4,C_5 = -0.0481, -1.1562, -0.6414, -0.473, and 0.434, respectively.

27. In Equation 24, the coefficient of variation of performance factor α, and hence the reliability of the design, is expressed as a function of the

coefficient of variation of the design parameters. W_t , W_A , W_{CBR} , and W_p are weighting factors which affect each of the corresponding design parameters. The factors are dependent upon the parameters P , A , and CBR but not upon t . A study of the weighting factors gives an insight into the effect of the design parameters.

28. Table 3 presents an evaluation of the weighting factors for a range of different values of variable A , CBR , and P . As shown, the weighting factor of A (tire contact area) is very small, and thus the effect of its variability on the reliability of the design is reduced. Since the weighting factor of t (the pavement thickness) is the largest and is always equal to 1, the effect of its thickness variability on performance variability is amplified. The fact that the weighting factors of wheel load P and subgrade CBR are equal indicates that the effect of their variabilities on pavement performance are equal. This conclusion is reasonable because in the CBR equation the performance factor α is proportional to the parameter CBR/P ; i.e., when one variable is changed, the performance can be kept unchanged if the other variable is changed in proportion. The conclusions derived from the weighting factors are consistent with those derived from the sensitivity analysis (paragraphs 24 and 25).

29. The pavements shown in Table 3 consist of values that are typical values in pavement design; i.e., for a 20-in. pavement built on a 4-CBR subgrade soil under a 30,000-lb single-wheel load, the design aircraft pass computed by the CBR equation is 800. Computations were made for extreme cases (underdesigned and overdesigned pavements) to check if the weighting factors would deviate from the values listed in Table 3; the results are presented in Table 4. For the case of underdesigning the pavement in which very low subgrade CBR and very heavy wheel load are used, the computed weighting factors in Table 4 are generally very close to those in Table 3, but for the case of overdesigning in which very high subgrade CBR and very light wheel load are used, the weighting factors of A , CBR , and P become very large, larger than that of t , which is always equal to 1. Fortunately, in the latter case when the subgrade is very strong and the load is very light, the design aircraft passes of the pavement will be very high, and the effect of design parameter variability on pavement performance in this case will not be significant, at least not as significant as at lower pass levels.

30. As shown in Table 3 and 4, the weighting factor of thickness t is

always equal to 1. If it is assumed that there is no variation in design parameters A , CBR , and P , Equation 24 can be written as

$$CV(\alpha) = CV(t) \tag{25}$$

Since $CV(\alpha) = \sigma_\alpha/\bar{\alpha}$, where σ_α is the standard deviation of the performance factor α and $\bar{\alpha}$ is the mean value of α , the following relation can be derived:

$$\bar{\alpha} = \frac{\sigma_\alpha}{CV(\alpha)} = \frac{\sigma_\alpha}{CV(t)}$$

or

$$\bar{\alpha} = \frac{\sigma_\alpha}{CV(t)} = \bar{t}\left(\frac{\sigma_\alpha}{\sigma_t}\right) \quad \text{for} \quad CV_A = CV_{CBR} = CV_P = 0 \tag{26}$$

where $\bar{t}$ is the mean pavement thickness. Equation 26 shows that for a given pavement in which the thickness t is selected and thus the performance factor α is computed, if there is no variation in other design parameters (i.e., P , CBR , and A), the standard deviation of α , i.e., σ_α , is directly proportional to the standard deviation of t , i.e., σ_t .

31. Equation similar to Equation 26 can also be written for other conditions:

$$\alpha = \frac{\sigma_\alpha}{\sqrt{W_A}\ CV(A)} = \frac{\bar{A}}{\sqrt{W_A}}\left(\frac{\sigma_\alpha}{\sigma_A}\right) \quad \text{for} \quad CV_t = CV_{CBR} = CV_P = 0 \tag{27}$$

$$\bar{\alpha} = \frac{\sigma_\alpha}{\sqrt{W_{CBR}}\ CV(CBR)} = \frac{\overline{CBR}}{\sqrt{W_{CBR}}}\left(\frac{\sigma_\alpha}{\sigma_{CBR}}\right) \quad \text{for} \quad CV_t = CV_A = CV_P = 0 \tag{28}$$

$$\bar{\alpha} = \frac{\sigma_\alpha}{\sqrt{W_P}\ CV(P)} = \frac{\bar{P}}{\sqrt{W_P}}\left(\frac{\sigma_\alpha}{\sigma_P}\right) \quad \text{for} \quad CV_t = CV_A = CV_{CBR} = 0 \tag{29}$$

where W_A , W_{CBR} , and W_P are weighting factors shown in Equation 24. As observed in Equations 27 through 29, the relationship between the standard deviations of α , i.e., σ_α , and the standard deviation of other design parameters (i.e., σ_P , σ_{CBR} , and σ_A) is not as simple as that between σ_α and σ_t . Also these equations show that the weighting factors W_A , W_{CBR} , and W_P are independent upon the thickness t .

32. In the discussion of reliability analysis in Part II, a procedure

was given to evaluate the reliability level of the design. The curves plotted for a particular design in Figures 2 and 3 were based on results shown in Table 1 for a thickness $t = 20$ in., as well as results computed for many other thicknesses. The coefficients of variation of design parameters P, A, CBR, and t were assumed to be 0.1. Figures 2 and 3 also show that for a given pavement thickness, the reliability of the design can be increased (or decreased) when the design airfield performance (aircraft passes) is decreased (or increased). At a given design aircraft pass level, the reliability of the design can be increased (or decreased) by increasing (or decreasing) the pavement thickness. For instance, for a 20-in. pavement, the predicted performance is 800 passes having an initial reliability level of 0.7 (which is assumed to be inherent in the CBR equation in this particular example). The reliability is reduced to 0.4 if the same pavement is designed to last 2,800 passes, but is increased to 0.8 if this pavement is designed to last only 620 passes. Another interpretation is that for a 20-in. pavement designed by the CBR equation, the chance of success that the pavement will last 800 passes is 70 percent (as the assumed reliability of the equation); the chance is reduced to 60 percent that the pavement will last 2,800 passes; and the chance is increased to 80 percent that the pavement will last 620 passes. Similar discussions may be presented for an assumed initial reliability value of 0.5 for the CBR equation as shown in Figure 3.

33. The relationships shown in Figure 2 are for the condition that the coefficients of variation for all of the four parameters (P, t, CBR, and A) are assumed to be 0.1. Computations were also made for other values of coefficient of variation, and the results are plotted in Figure 5 for the case of pavement thickness $t = 20$ in. For (very small) coefficients of variation of 0.01 (i.e., nearly no variations in design parameters and material variabilities), the reliability versus passes curve is almost a vertical line, indicating that the pavement performance can be predicted with very small chance of error. As the coefficients of variation are increased (i.e., variations in design parameters and material variabilities are larger), the reliability versus passes curve becomes flatter, suggesting that it is more difficult to accurately predict the pavement performance as the magnitude of error involved becomes larger.

34. The results presented in Figures 2 and 5 assume that all of the four design parameters have the same coefficient of variation. To study the

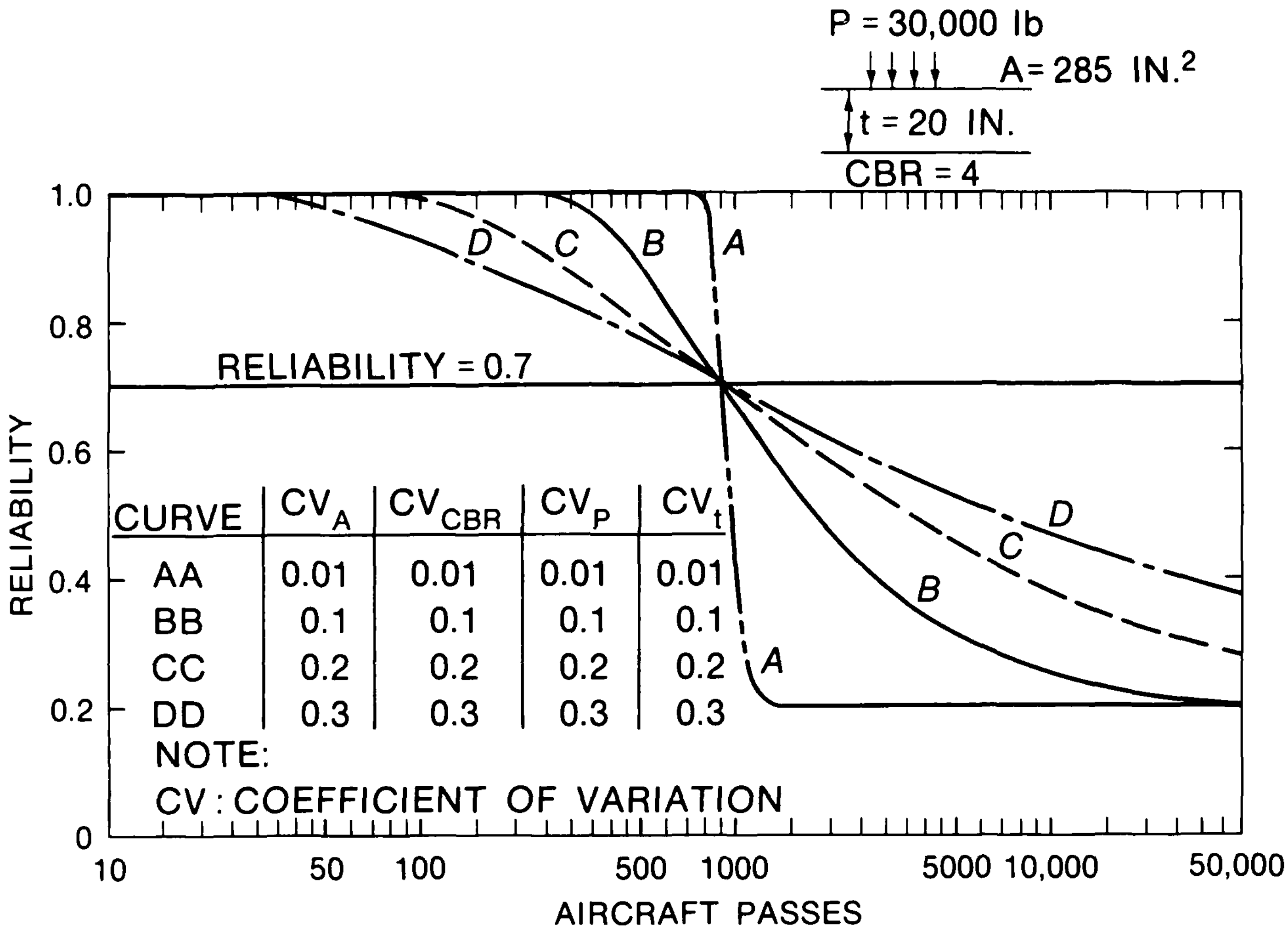

Figure 5. Relationships between reliability and passes for a given pavement with varying design parameter variabilities, reliability of the CBR equation equals 0.7

effect of each individual parameter, computations were made to vary only one parameter each time while the other three parameters were kept constant (i.e., the variations of the other three parameters were zero); the results are plotted in Figure 6 for an initial reliability value of 0.7. This figure shows that the pavement performance (aircraft passes) is least sensitive to the variation of the tire contact area A and is most sensitive to the variation of the pavement thickness t . The magnitude of the effects of the variations of the load P and the subgrade CBR on the pavement performance are practically the same. The conclusions derived from Figure 6 using the probabilistic approach are the same as those derived from the sensitivity analysis shown in Figure 4.

35. The significance of the results presented in Figure 6 may also be explained from another viewpoint by using the values listed in Table 5. Table 5 shows the ranges of wheel passes within +1 and -1 standard deviation of the α factor for four different cases. In each case, the coefficient of variation of one parameter is equal to 0.1 and that of the other three parameters are set to zero. The α factor computed from the CBR equation for the particular pavement is equal to 0.72.

36. Since the area within +1 and -1 standard deviation under a normal distribution curve is 0.68, the significance of the values shown in Table 5 can be explained in the following way. When only the variation of the tire contact area is accounted for ($CV_A = 0.1$), there is a 68 percent of chance* that the predicted performance falls within the range between 841 to 977 passes. When only the variation of the wheel load P is accounted for ($CV_P = 0.1$) and for the same 68 percent of chance, the predicted performance will fall within a range from 515 to 1,596 passes that indicates a larger variation. The same is true for the subgrade CBR variation ($CV_p = 0.1$). When only the variation of thickness t is accounted for ($CV_t = 0.1$), the range can be increased to between 339 to 2,429 passes for the same percent of chance. A larger range of predicted pavement performance indicates that the design has a greater amount of uncertainty.

Significance of the Analysis

37. The results presented in Figures 4 and 6 indicate that variations

* Sixty-eight percent is the percent of area covered within plus and minus one standard deviation under a normal distribution curve.

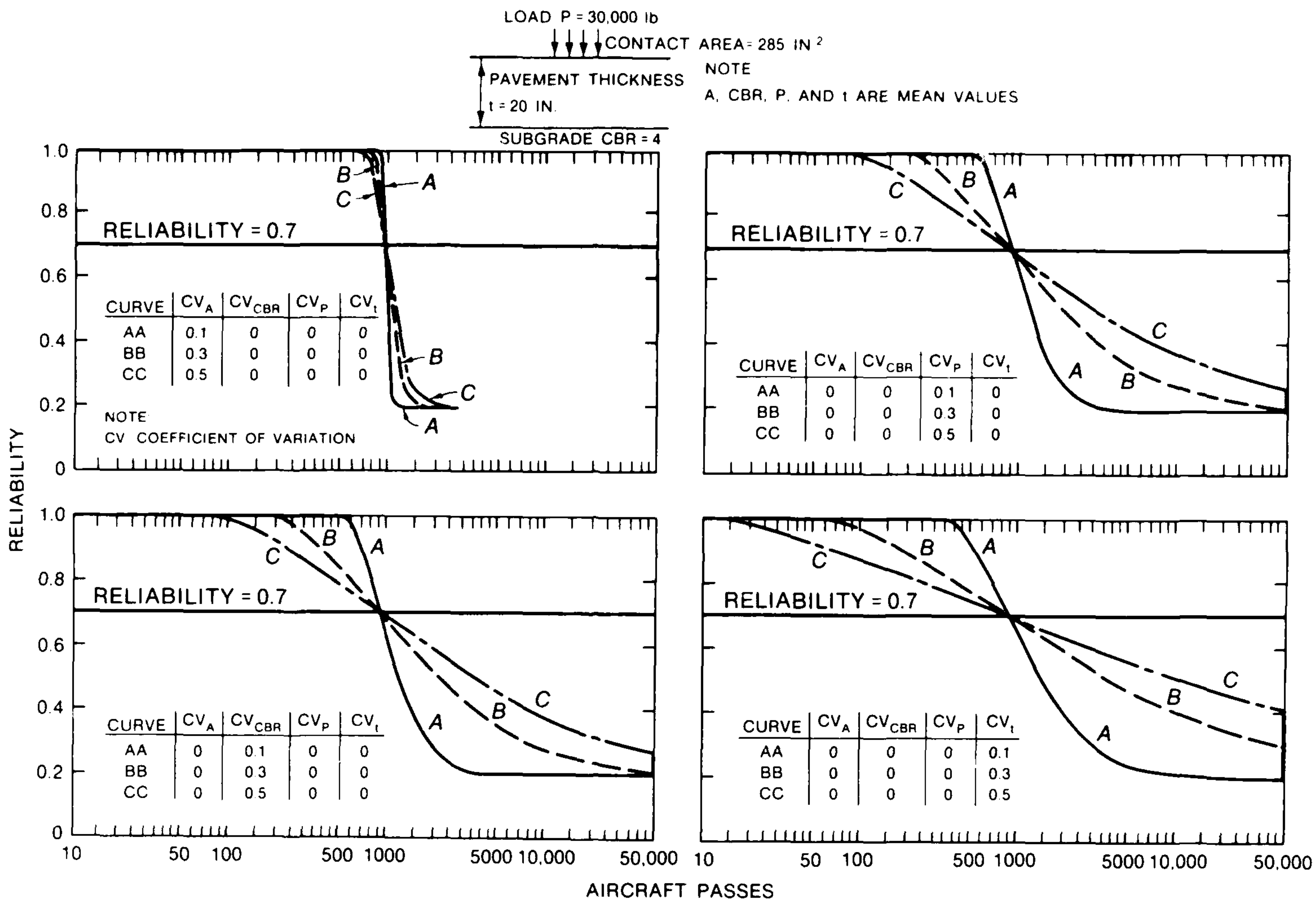

Figure 6. Reliability analysis for design parameter in the CBR equation, reliability of the CBR equation equals 0.7

of pavement thickness t have the largest effect on pavement performance, followed by the load P and subgrade CBR , and then the tire contact area A with the least effect. To control (or to limit the variation of) pavement performance, efforts should be made to control the pavement thickness and subgrade CBR during construction. Fortunately, it was found that the thickness variations in actual field constructions are not very large. Sherman (1971) measured pavement thicknesses in highway construction in California from 1962 to 1969. Table 6 presents the thickness variations for various pavement materials. Approximate standard errors obtained from the table by pooling the mean squares are shown below (Darter and Hudson 1973):

Material	Standard Error, in.	Number of Tests
Asphalt concrete	0.41	9,775
Cement-treated base	0.68	9,749
Aggregate base	0.79	8,053
Aggregate subbase	1.25	10,578

The average coefficients of variation for these pavement component layers are generally near or less than 10 percent. Nevertheless, more effort should be expected to reduce pavement thickness variation during construction.

38. Although the effect of the variation of subgrade CBR on pavement performance is not as large as that of pavement thickness, the actual variation of subgrade CBR in the field is known to be very large. The coefficient of variation may be expected to be 50 percent or more. More efficient construction methods and equipments should be used, and strict compaction and quality controls should be exercised in construction to reduce subgrade CBR variations, or alternatively reduce the mean CBR value.

39. The control of load variation is beyond the jurisdiction of pavement engineers. Since the variation of aircraft load has a large effect on pavement performance, the airfield operators should be informed and advised to limit aircraft overload cases.

40. Figures 4 and 6 show that the variation of aircraft tire contact area has the least effect on pavement performance. It should be pointed out that for a constant gross load P , the tire contact area A is controlled by the tire contact pressure p , as $pA = P$. As long as the aircraft load is under control, variations of tire contact area or tire inflation pressure have no significant effect on pavement performance.

Comparison of Results Computed by the Taylor Series Expansion and the Rosenblueth Method

41. The results presented in Figures 2, 5, and 6 were computed using Equations 4 and 8 (Taylor series expansion). The calculated values of the nonlinear terms in the equations, i.e., $(1/2)f''(\mu)\sigma_x^2$ in Equation 4 and $(1/4)[f''(\mu)]^2\sigma_x^4$ in Equation 8, proved to be very insignificant as compared with the calculated expected values (Equation 4) and variance (Equation 5). In other words, the nonlinear terms in the Taylor series expansion can actually be neglected.

42. Computations were made using the Rosenblueth method, and the results were compared with those computed using the Taylor series expansion. The comparison revealed that the differences computed using the two methods were small, the average difference in computed standard deviations was about 5 percent, but the difference became larger as the coefficients of variation of the design parameters increased. Table 7 shows the comparison between the two methods for a special design condition.

43. Since large differences in computed values are observed at larger coefficients of variation between the two methods, the question arises as which method yields better results at larger coefficients of variation. Since both methods are approximate in nature and since closed form solutions, with which solutions of approximate methods can be compared, are not available, conclusions cannot be drawn as to which method yields better results at greater variations of design parameters. Fortunately, Table 7 shows that differences computed from the two methods become large only when coefficients of variation of the parameters are greater than 0.3.

PART IV: CONCLUSIONS AND RECOMMENDATIONS

Conclusions

44. Based on results of the analysis of the CBR equation, the following conclusions were derived:

- a. Differences in computed results between the Taylor series expansion and the Rosenblueth method are small. Differences become larger when the variations of design parameters become very large.
- b. Prediction of pavement performance is most influenced by the variations of pavement thickness t and is least influenced by the variations of tire contact area A. The effects of variations of wheel load P and subgrade CBR are identical. The weighting factors for parameters t, CBR, P, and A, in general cases, are approximately 1, 0.34, 0.34, and 0.01 (Table 3), respectively.

Recommendations

45. To further improve the reliability of the design procedures, it is recommended that:

- a. Strict quality control should be exercised during construction to reduce variations of pavement thickness and subgrade CBR.
- b. The tire contact area can be discarded from the design parameters in the analysis of flexible pavements.

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Table 1

Computed Passes for Reliability Values of 0.5 and 0.7 for the CBR Equation

Reliability of the Design: Reliability of CBR Equation = 0.5	Reliability of the Design: Reliability of CBR Equation = 0.7	α Factor	Passes (from Figure 1)
0.5 + 0.5 = 1.0	0.7 + 0.5 > 1.0	$\alpha - 3\sigma_\alpha = 0.42$	16
0.5 + 0.49 = 0.99	0.7 + 0.49 > 1.0	$\alpha - 2.5\sigma_\alpha = 0.45$	31
0.5 + 0.48* = 0.98	0.7 + 0.48* > 1.0	$\alpha - 2\sigma_\alpha = 0.52$	61
0.5 + 0.43 = 0.93	0.7 + 0.43 > 1.0	$\alpha - 1.5\sigma_\alpha = 0.57$	121
0.5 + 0.34* = 0.84	0.7 + 0.34* > 1.0	$\alpha - \sigma_\alpha = 0.62$	237
0.5 + 0.19 = 0.69	0.7 + 0.19 = 0.89	$\alpha - 0.5\sigma_\alpha = 0.67$	464
0.5 + 0 = 0.5	0.7 + 0 = 0.70	$\alpha = 0.72$	910
0.5 - 0.19 = 0.31	0.7 - 0.19 = 0.51	$\alpha + 0.5\sigma_\alpha = 0.77$	1,782
0.5 - 0.34* = 0.16	0.7 - 0.34* = 0.36	$\alpha + \sigma_\alpha = 0.82$	3,491
0.5 - 0.43 = 0.07	0.7 - 0.43 = 0.27	$\alpha + 1.5\sigma_\alpha = 0.87$	7,032
0.5 - 0.48* = 0.02	0.7 - 0.48* = 0.22	$\alpha + 2\sigma_\alpha = 0.92$	14,635
0.5 - 0.49 = 0.01	0.7 - 0.49 = 0.21	$\alpha + 2.5\sigma_\alpha = 0.97$	30,456
0.5 - 0.5 = 0.0	0.7 - 0.5 = 0.2	$\alpha + 3\sigma_\alpha = 1.02$	63,379

Note: Pavement thickness $t = 20$ in., load $P = 30{,}000$ lb, subgrade CBR = 4, tire contact area $A = 285$ sq in., and $CV_P = CV_t = CV_{CBR} = CV_A = 0.1$.

* 0.34 and 0.48 are half of the area within plus and minus one and tw standard deviations, respectively, under a normal distribution curve.

Table 2

New Wheel Loads Used in the Sensitivity Analysis of CBR Equation

α* Factor	Multiplying Factor	New P**, lb
1.072	0.5	15,000
0.963	0.6	18,000
0.880	0.7	21,000
0.814	0.8	24,000
0.761	0.9	27,000
0.716	1.0	30,000
0.678	1.1	33,000
0.645	1.2	36,000
0.616	1.3	39,000
0.590	1.4	42,000
0.568	1.5	45,000

Note: Tire contact area $A = 285$ sq in., subgrade CBR = 4 , pavement thickness $t = 20$ in., and load $P = 30{,}000$ lb.

* Computed from Equation 23 based on new P .

** Obtained as the product of the multiplying factor and the 30,000 lb.

Table 3

Weighting Factors for Design Parameters, Pavement Thickness = 20 In.

	Pavement Conditions			Weighting Factors			
Case	A , sq in.	CBR	P , lb	W_A	W_{CBR}	W_P	W_t
1	125	4	30,000	0.00283	0.306	0.306	1
2	285	4	30,000	0.00582	0.332	0.332	1
3	550	4	30,000	0.00768	0.345	0.345	1
4	285	2	30,000	0.00335	0.311	0.311	1
5	285	10	30,000	0.00905	0.354	0.354	1
6	285	4	10,000	0.01087	0.365	0.365	1
7	285	4	60,000	0.00335	0.311	0.311	1

Table 4

Weighting Factors for Abnormal Designs, Pavement Thickness = 20 In.

	Pavement Conditions			Weighting Factors			
Case	A , sq in.	CBR	P , lb	W_A	W_{CBR}	W_P	W_t
1*	285	2	60,000	0.00088	0.281	0.281	1
2*	285	1	100,000	0.00087	0.221	0.221	1
3*	125	1	100,000	0.00484	0.185	0.185	1
4**	285	20	15,000	0.30028	1.098	1.098	1
5**	125	40	10,000	1.06520	2.347	2.347	1

* Underdesigned pavements.
** Overdesigned pavements.

Table 5

Performance Variations as Functions of Variabilities of Input Variables

Variabilities				Standard Deviation	α Values		Passes for	
CV_A	CV_{CBR}	CV_P	CV_t	of α , σ_α	$\alpha - \sigma_\alpha$	$\alpha + \sigma_\alpha$	$\alpha - \sigma_\alpha$	$\alpha + \sigma_\alpha$
0.1	0	0	0	0.005	0.71	0.72	841	977
0	0.1	0	0	0.041	0.67	0.76	515	1,596
0	0	0.1	0	0.041	0.67	0.76	515	1,596
0	0	0	0.1	0.072	0.64	0.79	339	2,429

Note: Load P = 30,000 lb, pavement thickness t = 20 in., subgrade CBR = 4, tire contact area A = 285 sq in., α = 0.72 computed from the CBR equation.

Table 6

Thickness Measurement Variations*

Year	Mean Deviation from Planned Material	Standard Thickness, ft	Number of Deviation	Measurements
1962	Asphalt concrete	+0.02	0.03	823
	Cement-treated base	+0.02	0.06	934
	Aggregate base	+0.00	0.07	1,149
	Aggregate subbase	0.00	0.08	1,037
1963	Asphalt concrete	+0.01	0.03	1,327
	Cement-treated base	+0.02	0.06	1,173
	Aggregate base	0.00	0.06	1,310
	Aggregate subbase	0.00	0.09	1,183
1964-1965	Asphalt concrete	+0.02	0.03	1,760
	Cement-treated base	+0.02	0.05	2,187
	Aggregate base	0.00	0.06	1,285
	Aggregate subbase	+0.02	0.10	1,922
1966	Asphalt concrete	+0.02	0.04	1,569
	Cement-treated base	0.00	0.06	1,569
	Aggregate base	0.00	0.07	1,272
	Aggregate subbase	+0.03	0.12	1,833
1967	Asphalt concrete	+0.01	0.03	1,838
	Cement-treated base	0.00	0.06	1,412
	Aggregate base	+0.01	0.07	1,134
	Aggregate subbase	+0.03	0.11	1,887
1968	Asphalt concrete	+0.02	0.04	1,135
	Cement-treated base	+0.01	0.05	1,156
	Aggregate base	+0.01	0.06	828
	Aggregate subbase	+0.01	0.10	1,526
1969	Asphalt concrete	+0.02	0.04	1,323
	Cement-treated base	+0.01	0.06	1,318
	Aggregate base	+0.02	0.07	1,075
	Aggregate subbase	+0.02	0.11	1,370

* From Sherman (1971).

Table 7

Comparison Between Computed Values of the Taylor and Rosenblueth Methods

Coefficients of Variation $CV_T = CV_{CBR} = CV_A = CV_P$	Computed Standard Deviation of the α Factor		Difference in Percent
	Taylor	Rosenblueth	
0.01	0.00924	0.00980	5.7
0.05	0.04623	0.04892	5.5
0.10	0.09238	0.09784	5.6
0.20	0.18412	0.19777	6.9
0.30	0.27453	0.30450	9.8
0.40	0.36293	0.42470	14.5
0.50	0.44862	0.57005	21.3

Note: Pavement thickness $t = 20$ in., subgrade CBR = 4 , tire contact area $A = 285$ sq in. and load $P = 30{,}000$ lb.

APPENDIX A. EXPECTATION AND VARIANCE OF A RANDOM VARIABLE*

1. The expectation of a discrete random variable x, denoted by $E(x)$ or simply μ_x, is defined as

$$E(x) = \sum_{\text{all } x_i} x_i f(x_i) \tag{A1}$$

where x_i represents all possible values of the random variable x and $f(x_i)$ is the probability-distribution (or probability-density) function which assigns the corresponding probability to each x_i.

2. The variance of x, denoted by $V(x)$ or σ_x^2, is defined as

$$V(x) = E\left[(x - \mu_x)^2\right] = \sum_{\text{all } x_i} (x_i - \mu_x)^2 f(x_i) \tag{A2}$$

the expectation and variance of x can also be defined as the first and second moments of x as explained below.

3. If x_1, x_2, ..., x_N are N values of a random variable x, *the k^{th} moment of x about the origin, $E(x^k)$ is defined as*

$$E(x^k) = \frac{x_1^k + x_2^k + \ldots + x_N^k}{N} \tag{A3}$$

The first $(k = 1)$ moment, $E(x^1)$, is the expected value of x.

4. If x_1, x_2, ..., x_M occur with frequencies of f_1, f_2, ..., f_m, respectively,

$$E(x^k) = \bar{x}_k = \frac{f_1 x_1^k + f_2 x_2^k + \ldots + f_M x_M^k}{N} \tag{A4}$$

where $N = \sum_{i=1}^{M} f_i$.

5. The k^{th} moment about the mean $\bar{x}_1$ or the k^{th} central moment of a random variable x is defined as

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* Readers may gain further information from Harr (1977). References cited in this Appendix are included in the References at the end of the main text.

of a random variable x is defined as

$$E\left[(x - \bar{x}_1)^k\right] = \sum_{i=1}^{N} \frac{\left(x_i - \bar{x}_1\right)^k}{N} \tag{A5}$$

The second central moment, $k = 2$, is the variance of x, $V(x)$. For grouped data with frequencies f_1, f_2, ..., f_M, $N = \sum_{i=1}^{M} f_i$,

$$E\left[(x - x_1)^k\right] = \frac{f_1\left(x_1 - \bar{x}_1\right)^k + f_2\left(x_2 - \bar{x}_1\right)^k + \ldots + f_M\left(x_M - \bar{x}_1\right)^k}{N} \tag{A6}$$

APPENDIX B: DERIVATION OF EXPECTED VALUE $E(\alpha)$ AND VARIANCE $V(\alpha)$ FOR THE CBR EQUATION

A. The modified CBR equation has the form

$$t = \alpha \left\{ \sqrt{A} \left[-0.0481 - 1.1562 \left(\log \frac{CBR \cdot A}{ESWL} \right) - 0.6414 \left(\log \frac{CBR \cdot A}{ESWL} \right)^2 + 0.473 \left(\log \frac{CBR \cdot A}{ESWL} \right)^3 \right] \right\} \quad \text{(B1)}$$

or,

$$\alpha = \frac{t}{\sqrt{A} \left[C_1 + C_2 \log VB + C_3 (\log VB)^2 + C_4 (\log VB)^3 \right]} \quad \text{(B2)}$$

where $VB = (CBR \cdot A)/ESWL$, and c_1 , c_2 , c_3 , and c_4 = -0.0481 , -1.1562, -0.6414, and -0.473, respectively.

B. For the sake of easy presentation and computer practice, Equation B2 can be further written as

$$\alpha = \frac{t}{D} \quad \text{(B3)}$$

where $D = \sqrt{A} \left[C_1 + C_2 \log VB + C_3 (\log VB)^2 + C_4 (\log VB)^3 \right]$.

C. The equation for solving the expected value $E(\alpha)$ and variance $V(\alpha)$ shown in Equations 4 and 8 are

$$E(\alpha) = \alpha\left(\bar{t}, \overline{CBR}, \bar{A}, \overline{ESWL}\right) + \frac{1}{2} \left(\sum_{i=1}^{4} \left. \frac{\partial^2 \alpha}{\partial x_i^2} \right|_{\text{all } \bar{x}_i} \right) \cdot \sigma_{x_1}^2 \quad \text{(B4)}$$

$$V(\alpha) = \left(\sum_{i=1}^{4} \left. \frac{\partial \alpha}{\partial x_i} \right|_{\text{all } \bar{x}_i} \cdot \sigma_{x_i} \right)^2 - \frac{1}{4} \left(\sum_{i=1}^{4} \left. \frac{\partial^2 \alpha}{\partial x_i^2} \right|_{\text{all } \bar{x}_i} \cdot \sigma_{x_i}^2 \right)^2 \quad \text{(B5)}$$

where

$\bar{E}$, $\overline{CBR}$, $\bar{A}$, $\overline{ESWL}$ = mean values of the parameters

$$\alpha(\bar{t}, \overline{CBR}, \bar{A}, \overline{ESWL}) = \frac{\bar{t}}{\left\{\sqrt{\bar{A}}\left[C_1 + C_2 \log \overline{VB} + C_3 (\log \overline{VB})^2 + C_4 (\log \overline{VB})^3\right]\right\}}$$

$$\sum_{i=1}^{4}\left(\frac{\partial \alpha}{\partial x_i}\bigg|_{\text{all } \bar{x}_i}\right)^2 \cdot \sigma_{x_i}^2$$

$$= \left[\frac{\partial\left(\frac{t}{D}\right)}{\partial t}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_t^2 + \left[\frac{\partial\left(\frac{t}{D}\right)}{\partial A}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_A^2$$

$$+ \left[\frac{\partial\left(\frac{t}{D}\right)}{\partial\ CBR}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_{CBR}^2 + \left[\frac{\partial\left(\frac{t}{D}\right)}{\partial\ ESWL}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_{ESWL}^2$$

$$\sum_{i=1}^{4}\left(\frac{\partial^2 \alpha}{\partial x_i^2}\bigg|_{\text{all } \bar{x}_i}\right)^2 \cdot \sigma_{x_i}^4$$

$$= \left[\frac{\partial^2\left(\frac{t}{D}\right)}{\partial t^2}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_t^4 + \left[\frac{\partial^2\left(\frac{t}{D}\right)}{\partial A^2}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_A^4$$

$$+ \left[\frac{\partial^2\left(\frac{t}{D}\right)}{\partial\ CBR^2}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_{CBR}^4 + \left[\frac{\partial^2\left(\frac{t}{D}\right)}{\partial\ ESWL^2}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_{ESWL}^4$$

where

σ_t , σ_{CBR} , σ_A , σ_{ESWL} = standard deviations of the parameters

D. The detail of the partial deviatives in Equations B4 and B5 are expressed as

$$\left[\frac{\partial\left(\frac{t}{D}\right)}{\partial t}\bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_t^2 = \frac{\sigma_t^2}{\bar{D}^2} \tag{B6}$$

$$\left[\frac{\partial\left(\frac{t}{D}\right)}{\partial A}\Bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_A^2$$

$$= \left\{\frac{1}{D^2}\left[-t\left(\frac{\partial D}{\partial A}\right)\right]_{\text{all } \bar{x}_i}\right\}^2 \cdot \sigma_A^2 = \left\{\frac{-t}{\bar{D}^2}\left[\frac{C_1 + 2C_2C_5}{2\sqrt{\bar{A}}}\right.\right.$$

$$+ \frac{C_2 + 4C_3C_5}{2\sqrt{\bar{A}}} \log \overline{VB} + \frac{C_3 + 6C_4C_5}{2\sqrt{\bar{A}}} \left(\log \overline{VB}\right)^2 \qquad \text{(B7)}$$

$$\left.\left. + \frac{C_4}{2\sqrt{\bar{A}}} \left(\log \overline{VB}\right)^3\right]\right\}^2 \cdot \sigma_A^2$$

$$\left[\frac{\partial\left(\frac{t}{D}\right)}{\partial\ CBR}\Bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_{CBR}^2$$

$$= \left\{\frac{-\bar{t}}{\bar{D}^2}\left[\frac{C_2C_5\sqrt{\bar{A}}}{\overline{CBR}} + \frac{2C_3C_5\sqrt{\bar{A}}}{\overline{CBR}} \log \overline{VB}\right.\right. \qquad \text{(B8)}$$

$$\left.\left. + \frac{3C_4C_5\sqrt{\bar{A}}}{\overline{CBR}} \left(\log \overline{VB}\right)^2\right]\right\}^2 \cdot \sigma_{CBR}^2$$

$$\left[\frac{\partial\left(\frac{t}{D}\right)}{\partial\ ESWL}\Bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_{ESWSL}^2$$

$$= \left\{\frac{-\bar{t}}{\bar{D}^2}\left[-\frac{C_2C_5\sqrt{\bar{A}}}{\overline{ESWL}} - \frac{2C_3C_5\sqrt{\bar{A}}}{\overline{ESWL}} \log \overline{VB}\right.\right. \qquad \text{(B9)}$$

$$\left.\left. - \frac{3C_4C_5\sqrt{\bar{A}}}{\overline{ESWL}} \left(\log \overline{VB}\right)^2\right]\right\}^2 \cdot \sigma_{ESWL}^2$$

$$\left[\frac{\partial^2\left(\frac{t}{D}\right)}{\partial t^2}\Bigg|_{\bar{t}}\right]^2 \cdot \sigma_t^4 = 0 \tag{B10}$$

$$\left[\frac{\partial^2\left(\frac{t}{D}\right)}{\partial A^2}\Bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_A^4 = -\bar{t}\left\{\frac{\partial}{\partial A}\left[\frac{C_1}{2\sqrt{A}\,D^2} + \frac{C_6}{D^2\sqrt{A}}\log \overline{VB} + \frac{C_2C_5}{D^2\sqrt{A}}\right.\right.$$

$$\left.\left. + \frac{C_7}{D^2\sqrt{A}}(\log \overline{VB})^2 + \frac{C_4}{2D^2\sqrt{A}}(\log \overline{VB})^3\right]\right\} \cdot \sigma_A^4$$

$$= \left\{-\bar{t}\left[\frac{C_1S_2}{2} + C_6S_2 \log \overline{VB} + \frac{C_5C_6}{2\bar{A}\sqrt{\bar{A}}\,\bar{D}^2} - 2C_2C_5\left(\bar{D}\sqrt{\bar{A}}\,S_1\right.\right.\right. \tag{B11}$$

$$\left. + \frac{\bar{D}^2}{4\sqrt{\bar{A}}}\right) + C_7S_2(\log \overline{VB})^2 + \frac{2C_7C_5}{\bar{D}^2\,\bar{A}\sqrt{\bar{A}}}\log \overline{VB}$$

$$\left.\left. + \frac{C_4S_2}{2}(\log \overline{VB})^3 + \frac{3C_4C_5}{2\bar{A}\sqrt{\bar{A}}\,\bar{D}^2}(\log \overline{VB})^2\right]\right\}^2 \cdot \sigma_A^4$$

$$\left[\frac{\partial^2\left(\frac{t}{D}\right)}{\partial\, CBR}\Bigg|_{\text{all } \bar{x}_i}\right]^2 \cdot \sigma_{CBR}^4$$

$$= \left\{-\bar{t}\left[C_2C_5S_4\sqrt{\bar{A}} + 2C_3C_5S_4\sqrt{\bar{A}}\log \overline{VB} + \frac{2C_3C_5^2\sqrt{\bar{A}}}{\overline{CBR}^2\,\bar{D}^2}\right.\right. \tag{B12}$$

$$\left.\left. + 3C_4C_5S_4\sqrt{\bar{A}}(\log \overline{VB})^2 + \frac{6C_4C_5\sqrt{\bar{A}}}{\overline{CBR}^2\,\bar{D}^2}\log \overline{VB}\right]\right\}^2 \cdot \sigma_{CBR}^4$$

$$\left[\frac{\partial^2\left(\frac{t}{D}\right)}{\partial \ ESWL} \Bigg|_{all\ \bar{x}_i} \right]^2 \cdot \sigma^4_{ESWL}$$

$$= \Bigg\{ -\bar{t} \Bigg[-C_2C_5S_6\sqrt{\bar{A}} - 2C_3C_5S_6\sqrt{\bar{A}} \log \overline{VB} - \frac{2C_3C_5^2\sqrt{\bar{A}}}{\bar{D}^2\ \overline{ESWL}^2} \qquad \text{(B13)}$$

$$- 3C_4C_5S_6\sqrt{\bar{A}} (\log \overline{VB})^2 + \frac{6C_4C_5^2\sqrt{\bar{A}}}{\bar{D}^2\ \overline{ESWL}^2} \log \overline{VB} \Bigg] \Bigg\}^2 \cdot \sigma^4_{ESWL}$$

where

$C_5 = \log_{10} e = 0.473429448$

$C_6 = 0.5C_2 + 2C_3C_5$

$C_7 = 0.5C_3 + 3C_4C_5$

$\bar{D} = \sqrt{\bar{A}} \left[C_1 + C_2 \log \overline{VB} + C_3 (\log \overline{VB})^2 + C_4 (\log \overline{VB})^3 \right]$

$$\overline{VB} = \frac{\overline{CBR}\ \bar{A}}{\overline{ESWL}}$$

$$S_1 = \frac{C_1}{2\sqrt{\bar{A}}} + \frac{(C_2 + 4C_3C_5)}{2\sqrt{\bar{A}}} \log \overline{VB} + \frac{C_2C_5}{\sqrt{\bar{A}}} + \frac{C_3}{2\sqrt{\bar{A}}} (\log \overline{VB})^2$$

$$+ \frac{C_4}{2\sqrt{\bar{A}}} (\log \overline{VB})^3 + \frac{3C_4C_5}{\sqrt{\bar{A}}} (\log \overline{VB})^2$$

$$S_2 = \frac{-1}{\bar{A}\ \bar{D}^3} \left(\frac{\bar{D}}{2\sqrt{\bar{A}}} + 2\bar{S}_1\sqrt{\bar{A}} \right)$$

$$S_3 = \frac{C_2C_5\sqrt{\bar{A}}}{\overline{CBR}} + \frac{2C_3C_5\sqrt{\bar{A}}}{\overline{CBR}} \log \overline{VB} + \frac{3C_4C_5\sqrt{\bar{A}}}{\overline{CBR}} (\log \overline{VB})^2$$

$$S_4 = \frac{-\left(\bar{D} + 2S_3\ \overline{CBR}\right)}{\overline{CBR}^2\ \bar{D}^3}$$

$$S_5 = \frac{-C_2C_5\sqrt{\bar{A}}}{\overline{ESWL}} - \frac{2C_3C_5\sqrt{\bar{A}}}{\overline{ESWL}} \log \overline{VB} - \frac{3C_4C_5\sqrt{\bar{A}}}{\overline{ESWL}} (\log \overline{VB})^2$$

$$S_6 = \frac{-\left(\bar{D} + 2S_5\ \overline{ESWL}\right)}{\bar{D}^3\ \overline{ESWL}^2}$$

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